Coherent structure extraction of turbulent jets for noise control

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goal = a physical understanding of the flow mechanisms enabling jet noise control
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Coherent structure analysis

- Coherent vortex extraction (CVE)
- Proper orthogonal decomposition (POD)
- Most observable decomposition (MOD)

Dynamic and stochastic modelling

- Galerkin models (GM)

Aeroacoustics

- CAA hypotheses
- Actuation design
- Control design
Jet flow data base

3000 LES/CAA snapshots of an incompressible jet ($Ma=0$) at $Re_D=3600$ over 300 convective time units

30D

76D

76 far-field sensors

[Diagram showing the flow field with sensor locations and time series data]
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Coherent vortex extraction (CVE)

orthogonal wavelet decomposition of vorticity snapshots

\[ \omega(x) = \sum_{\lambda \in \Lambda} \tilde{\omega}_\lambda \psi_\lambda(x) \]

with Coiflet 12 wavelets \( \psi_\lambda \)

CVE by thresholding of \( \tilde{\omega}_\lambda \)'s and reconstruction

**total vorticity** \( \omega = \)

**coherent vorticity** \( \omega_C + \)

**incoherent vorticity** \( \omega_I \)

enstrophy

\[ Z(\omega) = \int_{\Omega} \omega \cdot \omega \, dy = Z(\omega_C) + Z(\omega_I) \]

Figures display isosurfaces of the vorticity magnitude.
## CVE results

### Enstrophy Resolution

![Enstrophy Resolution Graph](image1)

### Enstrophy Spectra

![Enstrophy Spectra Graph](image2)

### PDF of Vorticity

![PDF of Vorticity Graph](image3)

### Table: Coherent and Incoherent Components

<table>
<thead>
<tr>
<th></th>
<th>N [%]</th>
<th>Z [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Coherent</td>
<td>4.75</td>
<td>91.35</td>
</tr>
<tr>
<td>Incoherent</td>
<td>95.25</td>
<td>8.65</td>
</tr>
</tbody>
</table>

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*Note: The PDF of vorticity graph shows a Gaussian fit.*
CVE examination of a CAA hypothesis

"Jet noise is generated from local coherent structures."

Helmholtz-Hodge decomposition of flow fields

incompressible space = 'fluxless knots' + 'curly gradients'

\[ u = u_\omega + \nabla \xi \]

where \( u_\omega = u_\omega(\omega) = \nabla \times \varphi \) employing the solution of problem \( \Delta \varphi = -\omega \) with homogeneous Dirichlet BC

CAA with \( u_\omega = u_\omega(\omega) \) and CAA with \( u_\omega = u_\omega(\omega_C) \)

In the left picture, results based on total (coherent) vorticity are represented by a blue line (red circles). In both pictures, the x-axis represents the streamwise locations of the aeroacoustic sensors.
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**Proper orthogonal decomposition (POD)**

**Hydrodynamic field**

\[ u'(x, t) \approx \sum_{i=1}^{N} a_i^u(t) u_i(x) \]

**Aerodynamic or aeroacoustic observable**

\[ q'(y, t) \approx \sum_{i=1}^{M} a_i^q(t) q_i(y) \]

**Most efficient resolution of hydrodynamic fluctuations**

\[ Q^\Omega (u') = \left\langle \int_{\Omega} u' \cdot u' \, dx \right\rangle \]

**Most efficient resolution of aerodynamic or aeroacoustic goal functionals**

\[ Q^\Gamma (q') = \left\langle \int_{\Gamma} q' \cdot q' \, dy \right\rangle \]

\[ \langle f \rangle := \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f \, dt \]
Definition of POD modes

POD modes \( u_i \) = critical points of maximum problem

\[
\max_{\varphi \in S^u} \frac{\left\langle \left| \int_{\Omega} u' \cdot \varphi \, dx \right|^2 \right\rangle}{\int_{\Omega} \varphi \cdot \varphi \, dx}
\]

\( u_i \)'s fulfill Fredholm integral equation on \( S^u \)

\[
\int_{\Omega} \left\langle u'(x, t) \otimes u'(x', t) \right\rangle u_i(x') \, dx' = \lambda_i^u u_i(x)
\]

with the POD eigenvalues \( \lambda_1^u \geq \lambda_2^u \geq \ldots \lambda_N^u > 0 \),
representing the modally resolved \( Q^\Omega(u') \)
POD results of incompressible jet residuum

284 modes resolve 90% of total kinetic energy \( K_\Omega := \frac{1}{2} Q_\Omega \)

velocity field \( u \)

mode 1

mode 10

mode 100

Figures display isosurfaces of the \( x \)-components (bright: positive, dark: negative).
**Galerkin-modelling of the incompressible jet**

**Galerkin system** $\dot{\mathbf{a}} = f(\mathbf{a})$ ($N = 30$)

\[
\mathbf{u}(x, t) \rightarrow \partial_t \mathbf{u} = \frac{1}{Re} \Delta \mathbf{u} - \nabla (\mathbf{u} \cdot \mathbf{u}) - \nabla p
\]

\[
\mathbf{u}[N] = \sum_{i=0}^{N} a_i \mathbf{u}_i \rightarrow \frac{d a_i}{d t} = c_i + \sum_{j=0}^{N} l_{ij} a_j + \sum_{j,k=0}^{N} q_{ijk} a_j a_k
\]

**mode coefficients**

Navier-Stokes attractor is represented by dotted lines, Galerkin model by solid lines.
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hydrodynamic field

\[ u'(x, t) \approx \sum_{i=1}^{N} a_i^u(t) u_i(x) \]

\[ q'(y, t) \approx \sum_{i=1}^{M} a_i^q(t) q_i(y) \]

most efficient resolution of hydrodynamic fluctuations

most efficient resolution of aerodynamic or aeroacoustic goal functionals

\[ Q^\Omega (u') = \left\langle \int_{\Omega} u' \cdot u' \, dx \right\rangle \]

\[ Q^\Gamma (q') = \left\langle \int_{\Gamma} q' \cdot q' \, dy \right\rangle \]

\[ \langle f \rangle := \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f(t) \, dt \]
Most observable decomposition (MOD)

optimal resolution of correlated goal functional:

\[ u' \approx \sum_{i=1}^{L} a_i^A(t) u_i^A(x) \quad \leftarrow \quad \text{"MOD approximation"} \]

most efficient resolution of correlated goal functional

\[ Q^A(u') = \left\langle \int_{\Gamma} q'(u') \cdot q'(u') \, dy \right\rangle \]

How to model the mapping from \( u' \) to \( q' \)?
Definition of MOD modes

Linear mapping \( (\tau = \text{time delay}) \):
\[
q'(t + \tau) = C \ u'(t)
\]

"Most observable" MOD modes:

\[ q_1 \quad \rightarrow \quad u_1^A := C^{-1} q_1 \]

\[ q_2 \quad \rightarrow \quad u_2^A := C^{-1} q_2 \]

\[ q_L \quad \rightarrow \quad u_L^A := C^{-1} q_L \]
Definition of MOD modes

ambiguity of $C^-$

⇒ add side constraints, addressed to control purposes

minimum principles
control perspective

flow attractor residual of MOD approximation
flow reconstruction

“energy” quantity $Q^\Omega(u')$
energy reduction causes reduction of the correlated goal functional

Dynamic observer design
Energy-based controller design

‘least-residual’ MOD
‘least-energetic’ MOD
Definition of MOD modes

each constraint defines 'observable' MOD subspace

\[ C^{-} S^q = C^{-} C S^u =: P S^u \]

with projection operator \( P \)

\[ \Phi_0 = P \Phi \]

\[ \Phi \]

state–space attractor

kernel of \( C^* C \)

range of \( C^* C \)

'observable' subspace
Definition of MOD modes

Each constraint defines 'observable' MOD subspace

\[ C^{-S^q} = C^{-C}S^u =: PS^u \] with projection operator \( P \)

**Diagram Details:**
- **Range of \( C^*C \):** The range of the operator \( C^*C \) is indicated by the green arrow pointing upwards.
- **Kernel of \( C^*C \):** The kernel of the operator \( C^*C \) is indicated by the blue arrow pointing to the right.
- **Vectors:**
  - \( \Phi_0 = P \Phi \) with \( P \) being the projection operator.
  - \( (I-P) \Phi \) indicates the part of \( \Phi \) not in the range of \( P \).
  - \( P^c \Phi \) and \( P^c \Phi_0 \) are shown in dashed lines, emphasizing their relationship to \( \Phi_0 \) and \( \Phi \).
Definition of MOD modes

Each constraint defines 'observable' MOD subspace
\[ C^{-S^q} = C^{-C}S^u =: P S^u \] with projection operator \( P \)

\[ \text{range of } C^*C \]

\[ \text{subspace of least attractor residuum} \]

\[ (I-P^Z)\Phi \]

\[ \text{kernel of } C^*C \]
Definition of MOD modes

for $P = \mathbb{P}^Z$ (LR-MOD) or $P = \mathbb{P}^C$ (LE-MOD):

MOD modes $u^A_i$ = critical points of maximum problem

$$\max_{\varphi \in \mathbb{P}S^u} \left\langle \left| \int_{\Omega} C u' \cdot C \varphi \, dx \right|^2 \right\rangle$$

$u^A_i$'s fulfill Fredholm integral equation on $\mathbb{P}S^u$

$$P \int_{\Omega} \left\langle u'(t) \otimes (C^* C u'(t)) (x') \right\rangle u^A_i(x') \, dx' = \lambda^q_i u^A_i$$

with the MOD eigenvalues $\lambda^q_1 \geq \lambda^q_2 \geq \ldots \lambda^q_L > 0$, representing the modally resolved $Q^A(u')$
MOD results of incompressible jet

MOD residual

33 modes resolve 90% of $Q^A(u')$.

⇒ System reduction by one order of magnitude !!!

least-residual MOD modes

mode 1

mode 2

mode 3

Figures display isosurfaces of the $x$-components (bright: positive, dark: negative).
Galerkin-model of a thin layer

Galerkin system $\dot{a} = f(a)$
describing flow dynamics
(thin layer around plane $z = 0$)

modal TKE distribution

Navier-Stokes attractor is represented by
dotted lines, Galerkin model by solid lines.

POD coefficient $a_5(t)$

POD coefficient $a_{80}(t)$
Model-based jet noise control

suppression of 'loud' structures by penalisation of energy growth in the 'least-energetic' MOD subspace, employing two plasma actuators at the end of the potential core

evolution of level of aeroacoustic fluctuations

⇒ noise reduction by 2 dB in average

Natural attractor dynamics are represented by dashed lines, controlled model dynamics by solid lines.
Summary

- Spatially local coherent structures can be distilled from CVE, substantiating the CAA hypothesis of being the cause of jet noise generalisation.
- Coherent jet structures are extracted based on POD.
- Modes, most contributing to a linearly related, aero-acoustic observable, are identified by MOD, a generalisation of POD which is tailored for purposes of observer and noise control design.
- Via MOD, a dimension reduction by one order of magnitude is obtained against POD.
- Penalising the energy flow into the “loud” MOD subspace, capability of a model-based control for jet noise suppression is demonstrated.
POD of 2D cylinder wake flow ($\text{Re}=100$)

\[ u' = \sum_{i=1}^{8} a_i u_i \]

8-dimensional POD model

\[ \frac{da_i}{dt} = f_i(a_1, \ldots, a_8) \]

reproduces DNS!

Streamlines of the POD modes $u_i$ are shown for $i = 1, \ldots, 8$. 
MOD of 2D cylinder wake flow (Re=100)

observable=lift

observable=drag

Only one MOD mode resolves 100% fluctuations respectively of lift and drag.

Streamlines are shown. The grid unit is given by the cylinder diameter.